Suppression of decoherence by bath ordering

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Hamiltonian

Calculation Procedure

Simulation and Results

Discussion

Conclusion

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Extended Tessieri-Wilkie Model

Two-body open system (Bell state) in a bath of $m = N - 2$ spins with inner-coupling λ_{bb}:

$$|\psi_S(0)\rangle = 1/\sqrt{2}(|10\rangle + |01\rangle).$$

$$\begin{aligned}
H &= H_s + H_b + H_{sb} \\
H_s &= \frac{\omega_s}{2}\sigma_1^z + \beta\sigma_1^x + \frac{\omega_s}{2}\sigma_2^z + \beta\sigma_2^x + \lambda_{ss}\sigma_1^x\sigma_2^x \\
H_b &= \sum_{i=3}^{N}\frac{\omega_b}{2}\sigma_i^z + \beta\sum_{i=3}^{N}\sigma_i^x + \lambda_{bb}\sum_{i=3}^{N-1}\sum_{j=i+1}^{N}\sigma_i^x\sigma_j^x \\
H_{sb} &= \lambda_{sb}\sum_{i=3}^{N}(\sigma_1^x\sigma_i^x + \sigma_2^x\sigma_i^x)
\end{aligned}$$

Polarization correlation and Concurrence

Quantum polarization correlation $\rightarrow$ coherence for $\rho_s = \rho_{AB}$

$$\begin{aligned} C_{A,B}^{\alpha,\beta} &= \langle \sigma_A^\alpha \sigma_B^\beta \rangle - \langle \sigma_A^\alpha \rangle \langle \sigma_B^\beta \rangle \\ &= \mathrm{T}r(\rho_{AB}\sigma_A^\alpha \sigma_B^\beta) - \mathrm{T}r(\rho_A \sigma^\alpha)\mathrm{T}r(\rho_B \sigma^\beta) \\ \alpha, \beta &= x, y, z \end{aligned}$$

Concurrence $\rightarrow$ entanglement

$$Con(\rho_s) = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$, are the square roots of the eigenvalues of $\rho_s(\sigma^y \otimes \sigma^y)\rho_s^*(\sigma^y \otimes \sigma^y)$

Diagram of steps

$$\begin{cases} \rho(0) = |\varphi_s(0)\rangle\langle\varphi_s(0)|\rho_b(0) \\ \rho_b(0) = \frac{\exp(-H_b/T)}{Z} \end{cases} \Rightarrow \begin{cases} \rho_b(0) = \sum_{n=1}^{2^m} |\chi_n^b\rangle\omega_n\langle\chi_n^b| \\ \omega_n = \frac{e^{(-E_n/T)}}{Z} \\ Z = \sum_{n=1}^{2^m} e^{(-E_n/T)} \end{cases}$$

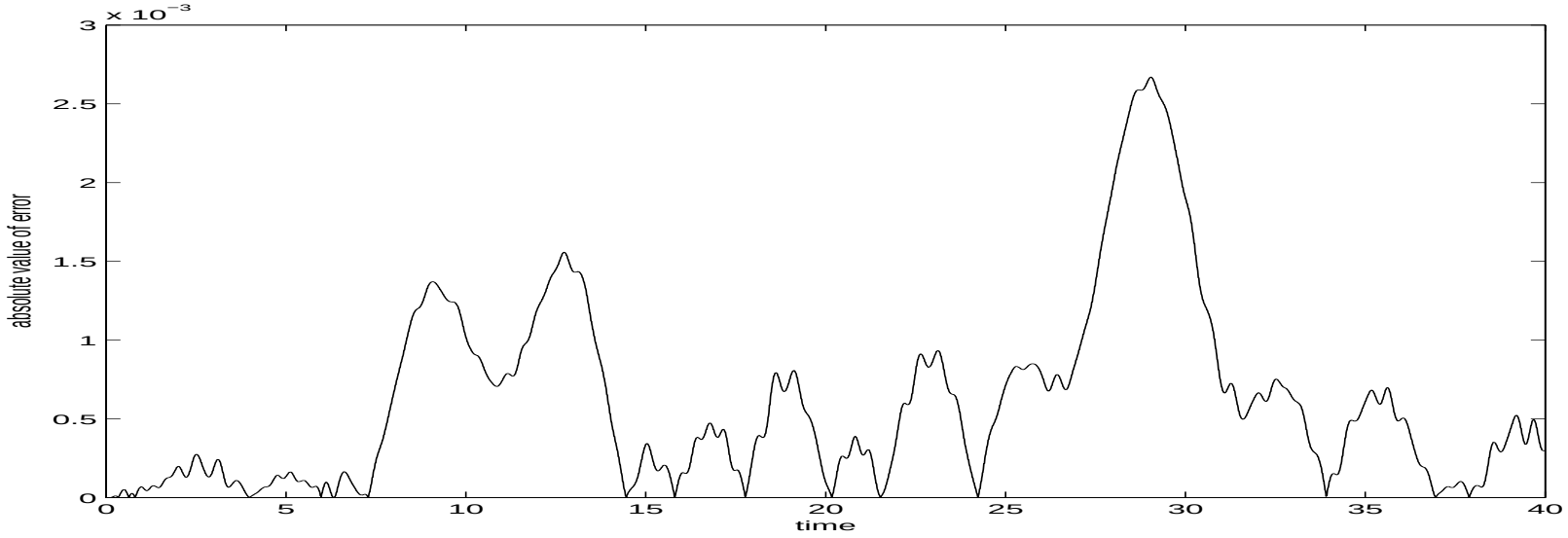
$$\downarrow \qquad\qquad \Downarrow$$

$$\begin{cases} \rho_s(t) = \mathrm{Tr}_b(\rho(t)) \\ \rho(t) = \sum_{n=1}^{M} \tilde{\omega}_n|\psi_n(t)\rangle\langle\psi_n(t)| \\ \psi_n(t) = \exp(-iHt)\psi_n(0) \\ \psi_n(0) = \varphi_s(0)\chi_n^b \end{cases} \Leftarrow \begin{cases} \rho_b(0) = \sum_{n=1}^{M} |\chi_n^b\rangle\tilde{\omega}_n\langle\chi_n^b| \\ \tilde{\omega}_n = \frac{e^{(-E_n/T)}}{Z} \\ Z = \sum_{n=1}^{M} e^{(-E_n/T)} \\ M \ll 2^m \end{cases}$$

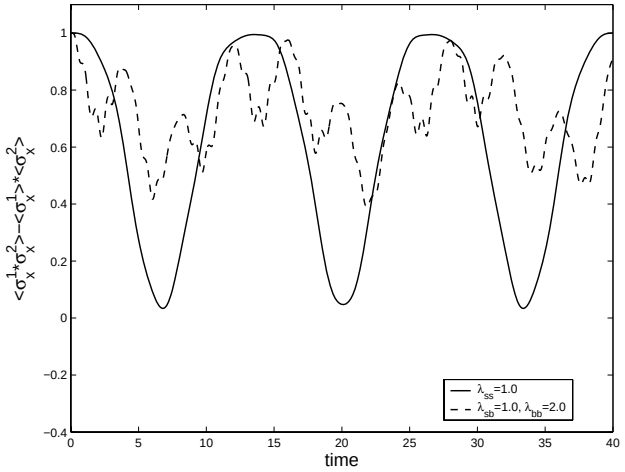
Approximation example

Most of the eigenstates could be omitted, for example, $(m = 8, \lambda_{sb} = 1.0, \lambda_{bb} = 4.0)$:

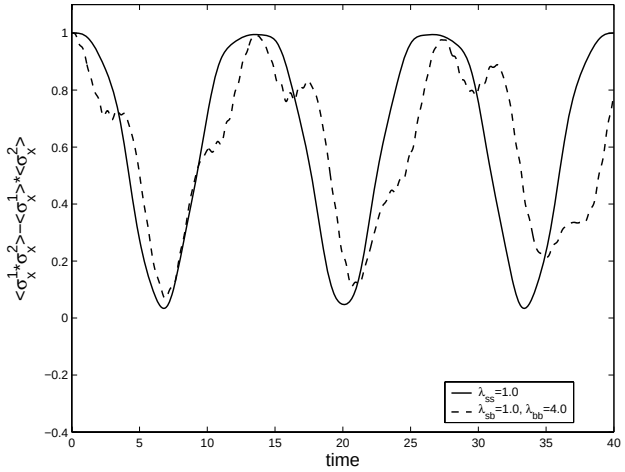
No. \ T	< 0.04	0.05	0.08	0.10	0.15	0.20
$M(\omega_n > 10^{-5})$	8	8	28	70	70	70
$M(\omega_n > 10^{-4})$	1	8	8	28	28	70
$M(\omega_n > 10^{-3})$	1	1	8	8	28	70



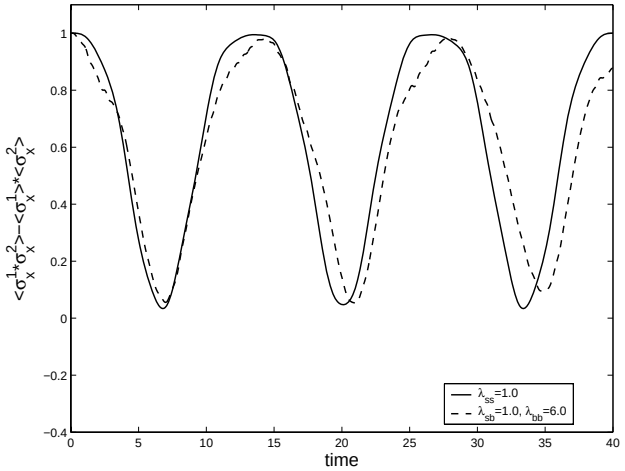
Polarization correlation along $\vec{x}$: $m = 4$



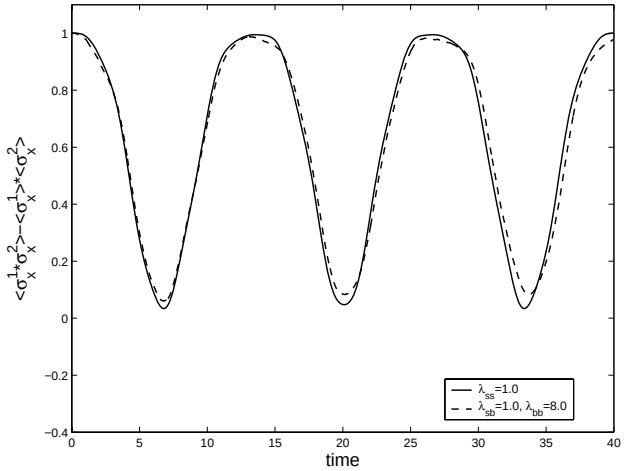
(a) $C_{1,2}^{xx}, \lambda = 2.0$



(b) $C_{1,2}^{xx}, \lambda = 4.0$



(c) $C_{1,2}^{xx}, \lambda = 6.0$



(d) $C_{1,2}^{xx}, \lambda = 8.0$

Polarization correlation along $\vec{y}$: $m = 4$

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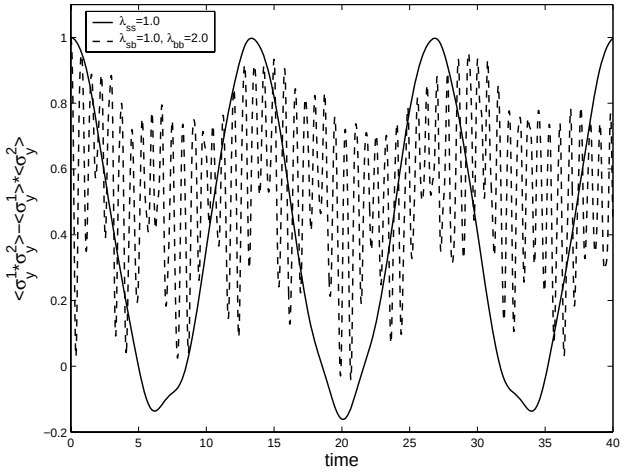
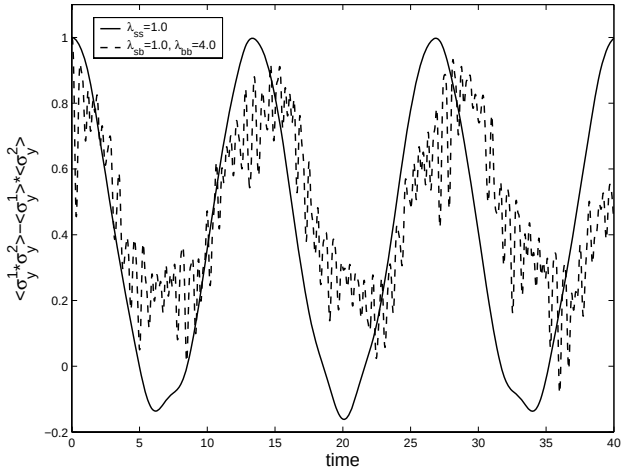
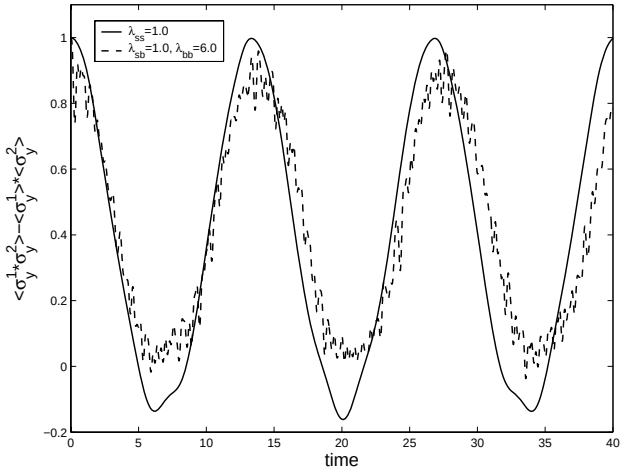


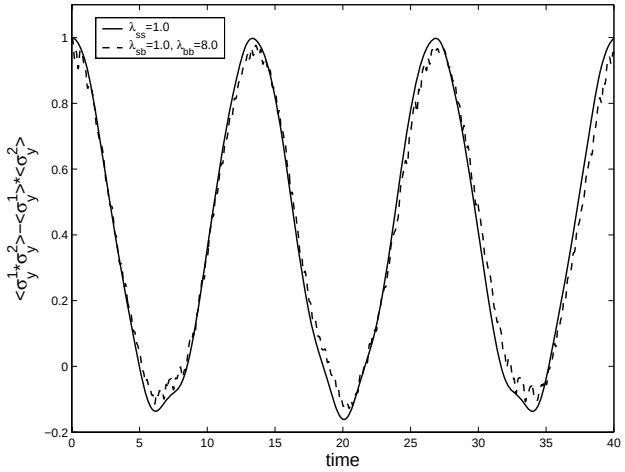
(e) $C^{yy}_{1,2}, \lambda = 2.0$



(f) $C^{yy}_{1,2}, \lambda = 4.0$



(g) $C^{yy}_{1,2}, \lambda = 6.0$



(h) $C^{yy}_{1,2}, \lambda = 8.0$

Polarization correlation along $\vec{z}$: $m = 4$

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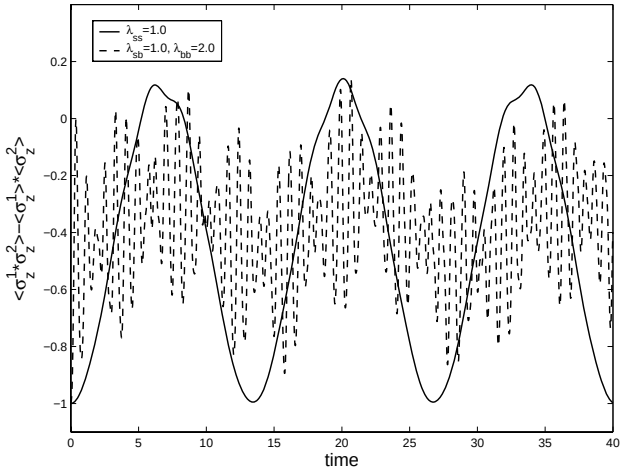
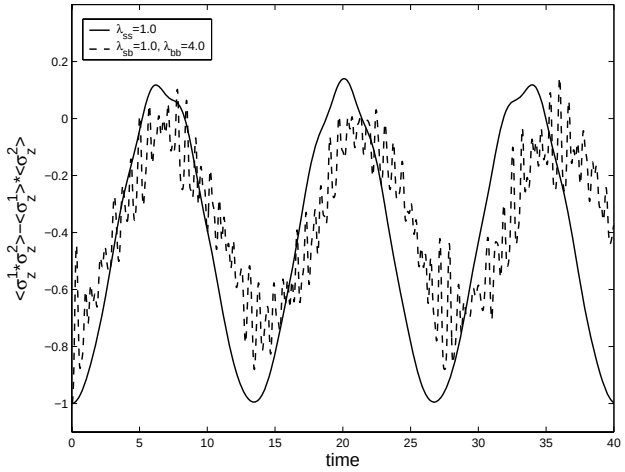
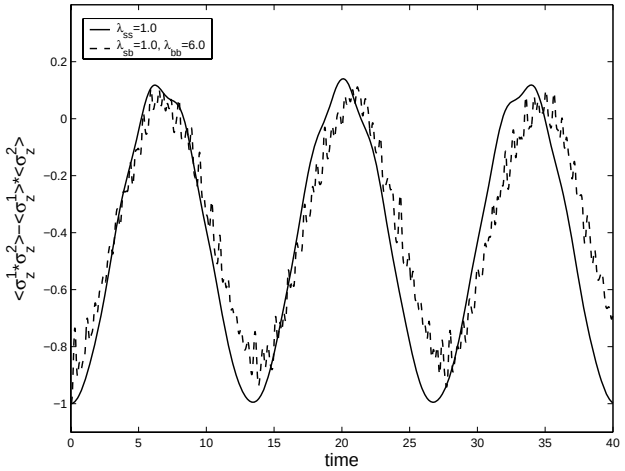


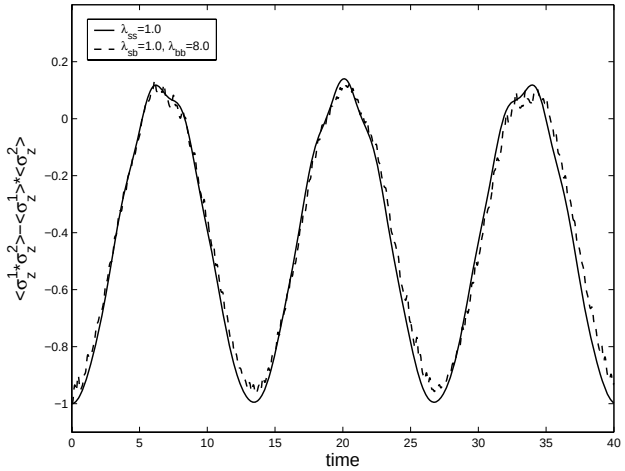
(i) $C^{zz}_{1,2}, \lambda = 2.0$



(j) $C^{zz}_{1,2}, \lambda = 4.0$

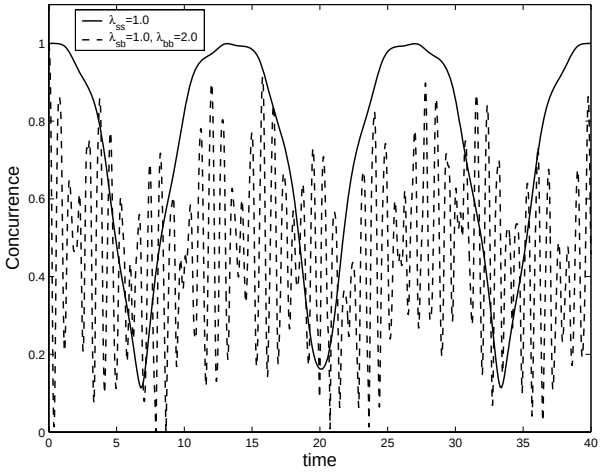


(k) $C^{zz}_{1,2}, \lambda = 6.0$

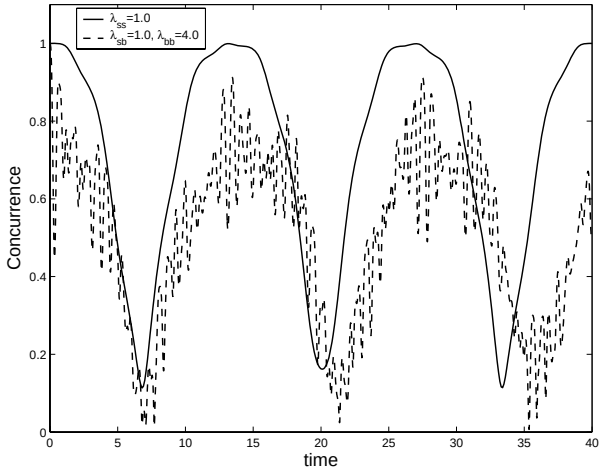


(l) $C^{zz}_{1,2}, \lambda = 8.0$

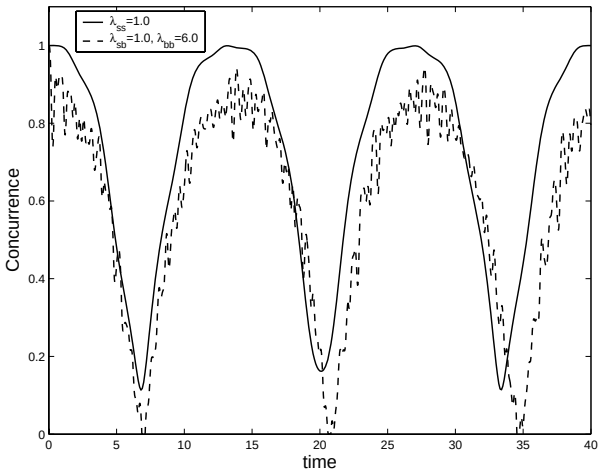
Concurrence: $m = 4$



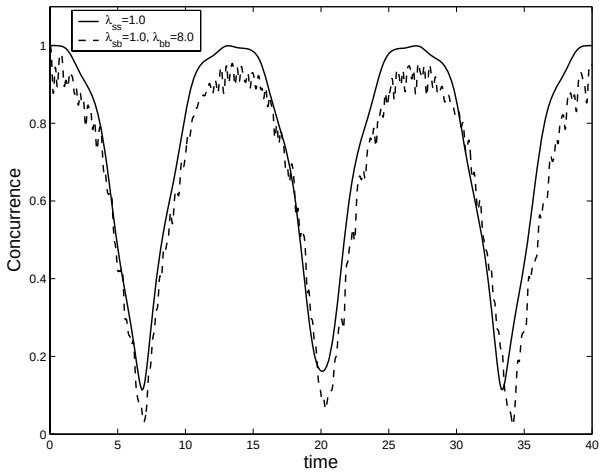
(m) $Con_{1,2}, \lambda = 2.0$



(n) $Con_{1,2}, \lambda = 4.0$

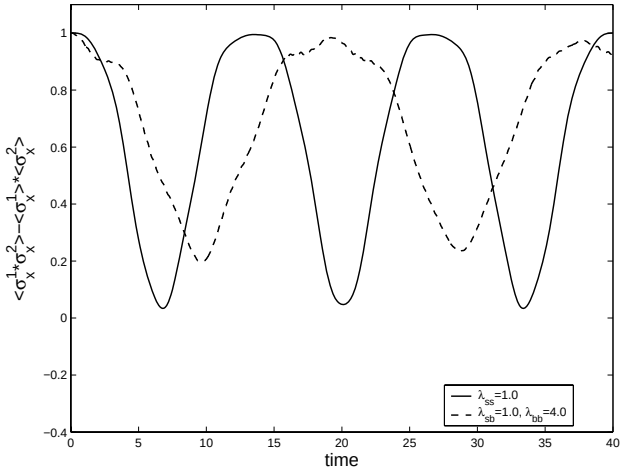


(o) $Con_{1,2}, \lambda = 6.0$

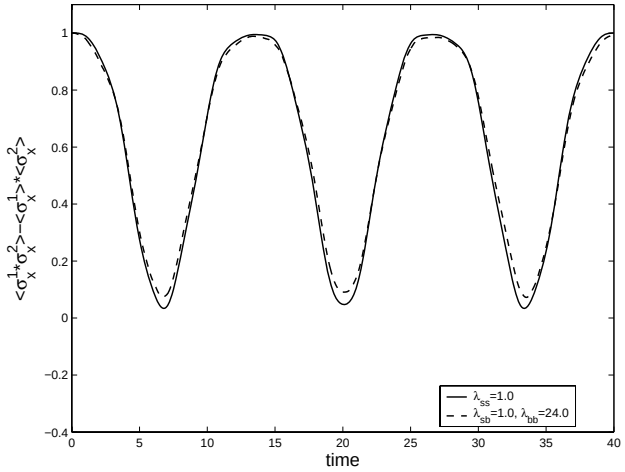


(p) $Con_{1,2}, \lambda = 8.0$

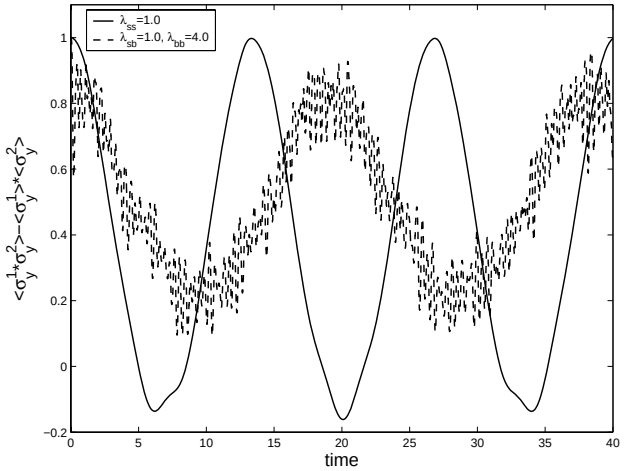
In a little bigger spin-bath: $m = 8$ I



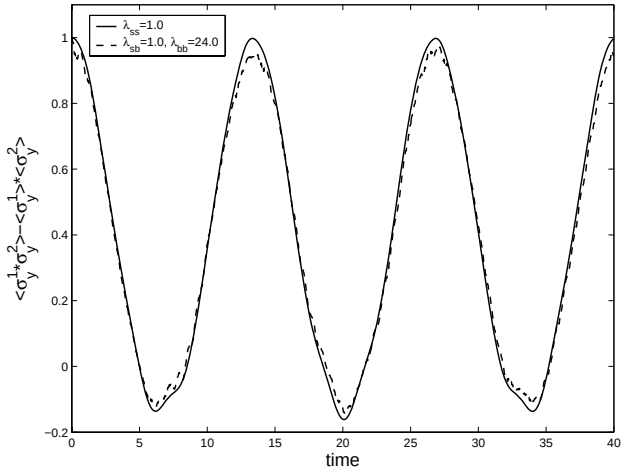
(q) $C_{1,2}^{xx}, \lambda = 4.0$



(r) $C_{1,2}^{xx}, \lambda = 24.0$



(s) $C_{1,2}^{yy}, \lambda = 4.0$



(t) $C_{1,2}^{yy}, \lambda = 24.0$

In a little bigger spin-bath: $m = 8$ II

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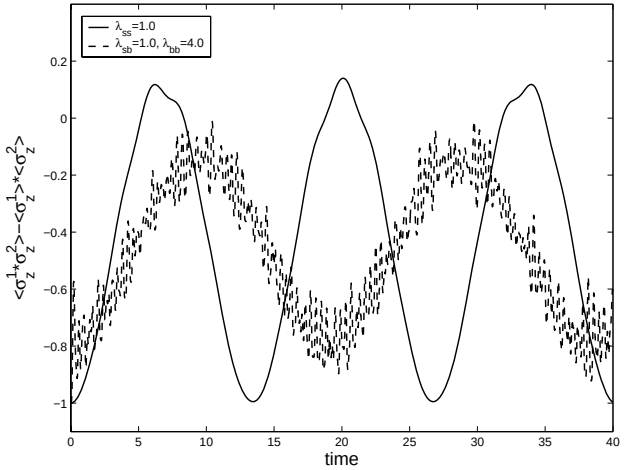
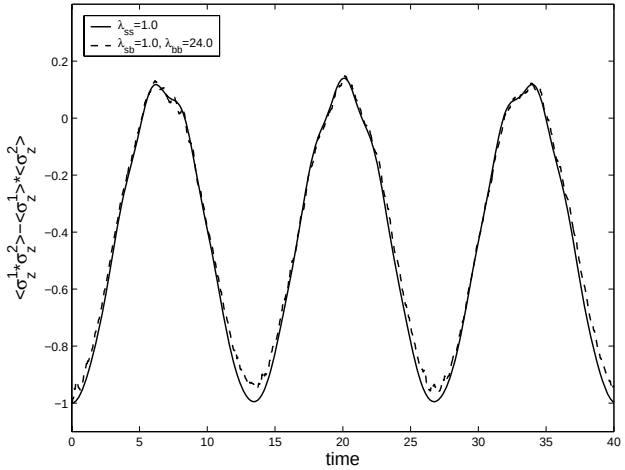
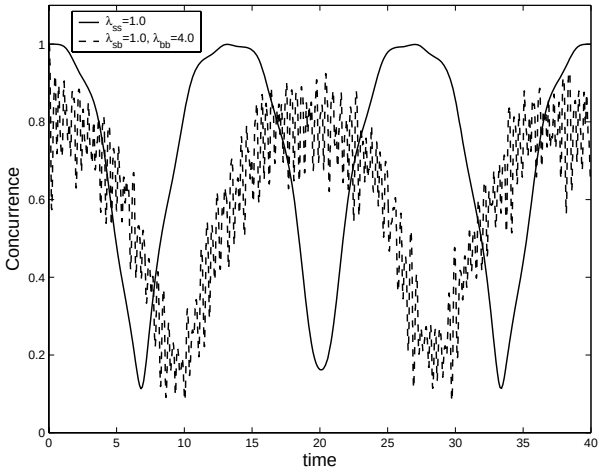


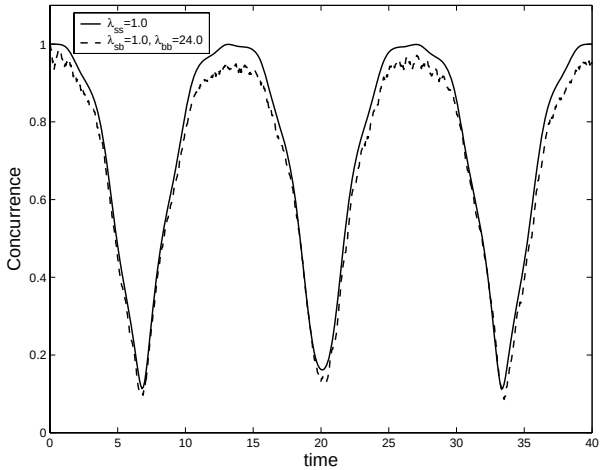
(u) $C^{zz}_{1,2}, \lambda = 4.0$



(v) $C^{zz}_{1,2}, \lambda = 24.0$



(w) $Con_{1,2}, \lambda = 4.0$



(x) $Con_{1,2}, \lambda = 24.0$

From representation of $\{\sigma^2, \sigma^z\}$ to $\{\sigma^2, \sigma^x\}$

Consider a system with 6 bath spins and two spins in the subsystem. By transformation:

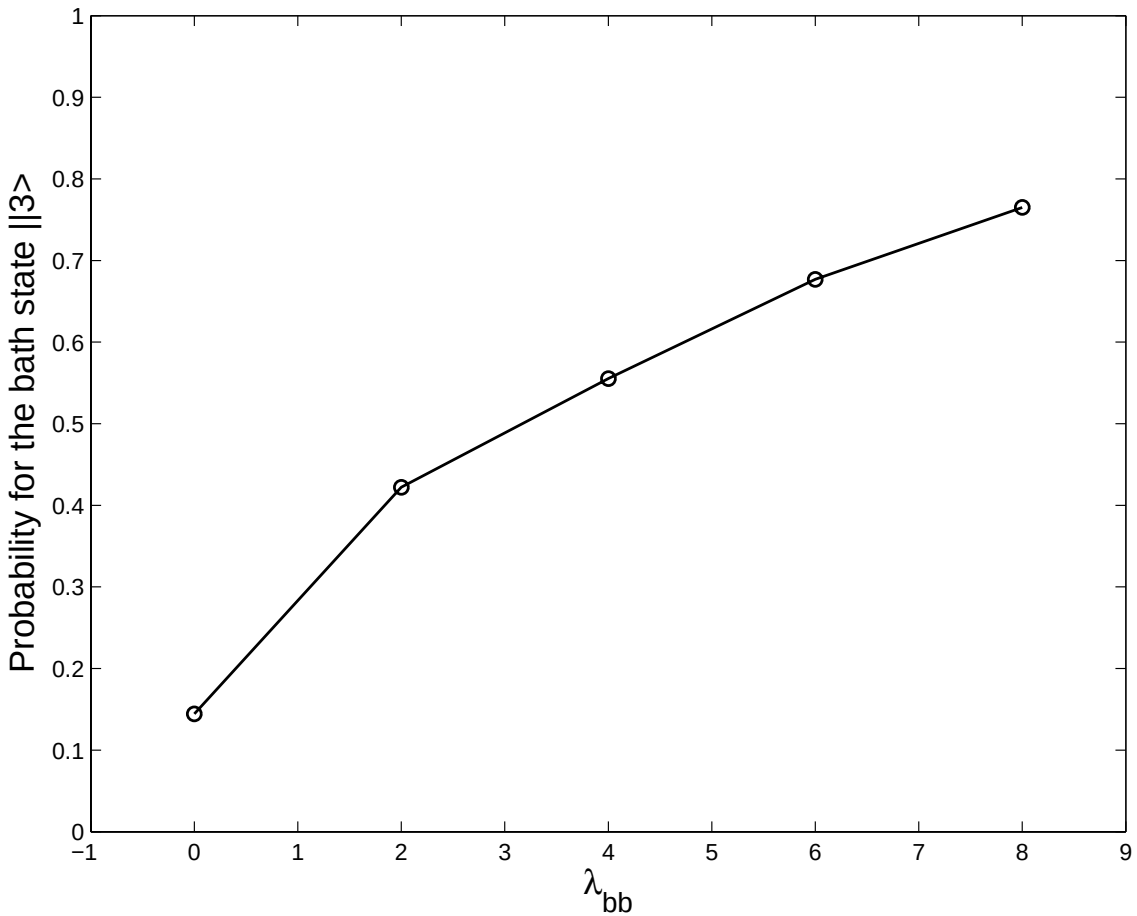
$$\begin{aligned} |\varphi^i\rangle_x &= U^{-1}|\varphi^i\rangle_z. \\ U &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \end{aligned}$$

The bath states can be expanded with the product state of the form $|i_1\rangle_x|i_2\rangle_x|i_3\rangle_x|i_4\rangle_x|i_5\rangle_x|i_6\rangle_x$.

There are $2^6 = 64$ product states, and for each product state, the subsystem can be in four states $|00\rangle_x$, $|10\rangle_x$, $|01\rangle_x$ and $|11\rangle_x$. We can use the number of "up" spins n to represent such states and denote it as $||n\rangle$.

Bath ordering

$$P(n) = \mathrm{Tr}_S \langle n || \rho(t) || n \rangle = \mathrm{Tr}_s \sum \omega_m \left| \langle n || \psi_m(t) \rangle \right|^2 .$$



Remarks

- Strong intra-bath coupling can suppress the decoherence and disentanglement of the subsystem spins, especially the initial state of which is $|\psi_S(0)\rangle = 1/\sqrt{2}(|01\rangle + |10\rangle)$.
- The suppression of decoherence effect is also depends on the number of bath spins, it is found that the more bath spins, the less effective of suppression of the decoherence.
- As the coupling strength increases, the bath spins are self organized to a ordered state in x direction, which could be proved by the probability that the bath state appears in a resonant antiferromagnetic ordered state. It is this ordering that brings the subsystem to the more coherent state.

Thank you!