

Dynamics of two qubits in a spin bath with anisotropic XY coupling

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The dynamics of two $1/2$ -spin qubits under the influence of a quantum Heisenberg XY -type spin-bath is studied. After the Holstein-Primakoff transformation, numerical polynomial scheme is used to give the time-evolution calculation of the center qubits initially prepared in a product state or a Bell state. Then the concurrence of the two qubits, the z -component moment of either of the subsystem spins, and the fidelity of the subsystem are shown; they exhibit sensitive dependence on the anisotropic parameter, the temperature, the coupling strength, and the initial state. It is found that (i) the larger the anisotropic parameter γ , the bigger the probability of maintaining the initial state of the two qubits; (ii) with increasing temperature T , the bath plays a stronger destroy effect on the dynamics of the subsystem, so does the interaction g_0 between the subsystem and the bath; and (iii) the time evolution of the subsystem is dependent on the initial state. The revival of the concurrence does not always mean the restoration of the state. Further, the dynamical properties of the subsystem should be judged by the combination of concurrence and fidelity.

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I. INTRODUCTION

Solid-state devices, in particular, ultrasmall quantum dots¹ with spin degrees of freedom embedded in nanostructured materials, compared with other physical systems such as ions in trap,² are more easily scaled up to large registers, and they can be manipulated by energy bias and tunneling potentials.³ The key building block of a quantum processor consists of two entangled quantum bits. Thus, the spin system is one of the most promising candidates for quantum computation owing to its long relaxation and decoherence times.³⁻⁵ However, the spin qubits are open systems for which it is impossible to avoid interactions with their environments.⁶⁻⁹ Finally, the states of the qubits will relax into a set of “pointer states” in the Hilbert space,¹⁰ and the entanglement between the spin qubits will also vanish. Yet, the entanglement is the most intriguing feature of quantum composite system and the vital resource for quantum computation and quantum communication.^{5,11} These are so-called decoherence and disentanglement processes. These two disadvantages will not be overcome until there is modeling of the surrounding environment or bath of the spin systems.

For solid-state spin nanodevices, the quantum noise mainly arises from the contribution of nuclear spins, which could be regarded as a spin environment. Recently, there have been lots of works devoted to studying the behavior of center spins under the strong non-Markovian influence of a spin bath.^{12,13} Paganelli *et al.* and Lucamarini *et al.* made use of perturbative expansion method¹⁴ and mean-field approximation¹⁵ to study the temporal evolution of entanglement pertaining to one qubit interacting with a thermal bath. They found entangled states with an exponential decay of the quantum correlation at finite temperature. Hutton and Bose¹⁶ investigated a star network of spins at zero temperature, in which all spins interact exclusively and continuously with a central spin through Heisenberg XX couplings of equal strength. Their work was advanced by Hamdouni *et al.*,¹⁷ who derived the exact reduced dynamics of a central two-

qubit subsystem in the same bath configuration, also studied the entanglement evolution of the central system. Yuan *et al.*⁹ used a novel operator technique to obtain the dynamics of the two coupled spins in quantum Heisenberg XY high symmetry spin model. The results of all the above works are very exciting. Yet, their methods are of some kinds of complex analytical derivations. In Ref. 9, their analytical results are dependent on some particular initial states and essentially the interaction between the spins in their model is isotropic. Here, we introduce a “half analytical and half numerical” method to solve such an open quantum system problem in an anisotropic Heisenberg XY model. The present model involves the Heisenberg XY interaction that has broad applications for various quantum information processing systems, such as quantum dots, cavity QED, etc.¹⁸⁻²¹ Also, our method is initial state independent.

In this paper, we study an open two-spin-qubit system in a spin bath of starlike configuration, which is similar to the cases studied in Refs. 9 and 17. However, the two qubits’ distance is far enough so that the direct coupling between them could be neglected. Then, we can concentrate on discussing the role of the bath in this model. First, we use Holstein-Primakoff transformation to reduce the model to an effective “spin-boson” Hamiltonian. Then, we apply a numerical simulation to obtain the reduced dynamics of the two-spin qubits. During our numerical calculation, there are no approximations assumed and the initial state of the subsystem (consisted by the two-spin qubits) can be arbitrary. It is well known that the concurrence is a measure of entanglement degree between two-spin qubits and the fidelity is also an important property that has been widely applied quantum coding theory.²² Thus, some results about these quantities in the thermal limit will be given in the latter part of this paper. The rest of this paper is organized as follows. In Sec. II, the model Hamiltonian and the operator transformation procedure are introduced. In Sec. III, we explain the numerical techniques about the evolution of the reduced matrix for the subsystem. Detailed results and discussions can be found in Sec. IV. We will conclude our study in Sec. V.

II. MODEL AND TRANSFORMATION

Consider a two-spin-qubit subsystem symmetrically interacting with bath spins via a Heisenberg XY interaction: both the subsystem and the bath are composed of spin-1/2 atoms. Every spin in the bath interacts with each of the two center spins of equal strength, similar to the cases considered in Refs. 8, 9, 16, and 23. The Hamiltonian for the total system is divided into three parts,

$$H = H_S + H_{SB} + H_B, \quad (1)$$

where

$$H_S = \mu_0(\sigma_{01}^z + \sigma_{02}^z), \quad (2)$$

$$H_{SB} = \frac{g_0}{2\sqrt{N}} \sum_{i=1}^N [(1 + \gamma)(\sigma_{01}^x \sigma_i^x + \sigma_{02}^x \sigma_i^x) + (1 - \gamma)(\sigma_{01}^y \sigma_i^y + \sigma_{02}^y \sigma_i^y)], \quad (3)$$

$$H_B = \frac{g}{2N} \sum_{i \neq j}^N [(1 + \gamma)\sigma_i^x \sigma_j^x + (1 - \gamma)\sigma_i^y \sigma_j^y]. \quad (4)$$

Here, H_S and H_B are the Hamiltonians of the subsystem and bath, respectively, and H_{SB} describes the interaction between them.^{8,9,24} μ_0 represents the coupling constant between a locally applied external magnetic field in the z direction and the spin qubit subsystem. γ , $-1 \leq \gamma \leq 1$ is the anisotropic parameter. When $\gamma=0$, it is of an isotropic case.⁹ In the following part of this paper, we only talk about cases with positive γ for the symmetry of the spin star structure. σ_{0i}^x , σ_{0i}^y , and σ_{0i}^z ($i=1,2$) are the operators of the qubit subsystem spins, respectively. By Pauli matrix, the operators read

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (5)$$

σ_i^x and σ_i^y are the corresponding operators of the i th atom spin in the bath. The indices i of the summation for the spin bath run from 1 to N , where N is the number of the bath atoms. g_0 is the coupling constant between the qubit subsystem spins and bath spins, whereas g is the coupling between the bath spins.

Using $\sigma^x = (\sigma^+ + \sigma^-)$ and $\sigma^y = -i(\sigma^+ - \sigma^-)$, we can rewrite Hamiltonians (3) and (4) as

$$H_{SB} = \frac{g_0}{\sqrt{N}} \left[\sum_{i=1}^N \sigma_i^+ (\gamma \sigma_{01}^+ + \sigma_{01}^-) + \sum_{i=1}^N \sigma_i^- (\sigma_{01}^+ + \gamma \sigma_{01}^-) + \sum_{i=1}^N \sigma_i^+ (\gamma \sigma_{02}^+ + \sigma_{02}^-) + \sum_{i=1}^N \sigma_i^- (\sigma_{02}^+ + \gamma \sigma_{02}^-) \right], \quad (6)$$

$$H_B = \frac{g}{N} \sum_{i \neq j}^N [\gamma(\sigma_i^+ \sigma_j^+ + \sigma_i^- \sigma_j^-) + (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+)]. \quad (7)$$

Substituting the collective angular momentum operators $J_\pm = \sum_{i=1}^N \sigma_i^\pm$ into Eqs. (6) and (7), we get

$$H_{SB} = \frac{g_0}{\sqrt{2j}} [J_+ (\gamma \sigma_{01}^+ + \sigma_{01}^-) + J_- (\sigma_{01}^+ + \gamma \sigma_{01}^-) + J_+ (\gamma \sigma_{02}^+ + \sigma_{02}^-) + J_- (\sigma_{02}^+ + \gamma \sigma_{02}^-)], \quad (8)$$

$$H_B = \frac{g}{2j} [\gamma(J_+ J_+ + J_- J_-) + (J_+ J_- + J_- J_+ - 2j)]. \quad (9)$$

where $j = N/2$. After the Holstein-Primakoff transformation,²⁵

$$J_+ = b^+ (\sqrt{2j - b^+ b}), \quad J_- = (\sqrt{2j - b^+ b}) b, \quad (10)$$

with $[b, b^+] = 1$ and in the thermodynamic limit (i.e., $N \rightarrow \infty$) at finite temperatures, the Hamiltonian [Eqs. (8) and (9)] can finally be written as

$$H_{SB} = g_0 [b^+ (\gamma \sigma_{01}^+ + \sigma_{01}^- + \gamma \sigma_{02}^+ + \sigma_{02}^-) + b (\sigma_{01}^+ + \gamma \sigma_{01}^- + \sigma_{02}^+ + \gamma \sigma_{02}^-)], \quad (11)$$

$$H_B = g [\gamma(b^{+2} + b^2) + 2b^+ b]. \quad (12)$$

The transformed Hamiltonian describes two qubits interacting with a single-mode thermal bosonic bath field, so the analysis of the model is just like a nontrivial problem in the field of cavity quantum electrodynamics.^{18,19} We note here that due to the transition invariance of the bath spins in our model, it is effectively represented by a single collective environment pseudospin J in Eq. (10). After the Holstein-Primakoff transformation and in the thermodynamic limit, this collective environment pseudospin could be considered a single-mode bosonic thermal field. The effect of this single-mode environment on the dynamics of the two qubits is interesting. In Sec. IV, we will show some results, for example, the revival behavior of the reduced density matrix or entanglement evolution of the subsystem spins. This can be used in real quantum information application.

III. NUMERICAL CALCULATION PROCEDURES

The initial density matrix of the total system is assumed to be separable, i.e., $\rho(0) = |\psi\rangle\langle\psi| \otimes \rho_B$. The density matrix of the spin bath satisfies the Boltzmann distribution, that is, $\rho_B = e^{-H_B/T}/Z$, where $Z = \text{Tr}(e^{-H_B/T})$ is the partition function, and the Boltzmann constant k_B has been set to 1 for simplicity. The density matrix $\rho(t)$ of the whole system can formally be derived by

$$\rho(t) = \exp(-iHt)\rho(0)\exp(iHt), \quad (13)$$

$$\rho(0) = \rho_S(0) \otimes \rho_B(0), \quad (14)$$

$$\rho_S(0) = |\psi(0)\rangle\langle\psi(0)|. \quad (15)$$

In order to find the density matrix $\rho(t)$, we follow the method suggested by Tessieri and Wilkie.²⁶ The thermal bath state $\rho_B(0)$ can be expanded with the eigenstates of the environment Hamiltonian H_B in Eq. (12),

$$\rho_B(0) = \sum_{m=1}^M |\phi_m\rangle\omega_m\langle\phi_m|, \quad (16)$$

$$\omega_m = \frac{e^{-E_m/T}}{Z}, \quad (17)$$

$$Z = \sum_{m=1}^M e^{-E_m/T}. \quad (18)$$

Here, $|\phi_m\rangle$, $m=1, 2, 3, \dots, M$, are the eigenstates of H_B and E_m the corresponding eigenenergies in increasing order. M is just the number of eigenstates considered in this summation. With this expansion, the density matrix $\rho(t)$ can be written as

$$\rho(t) = \sum_{m=1}^M \omega_m |\Psi_m(t)\rangle\langle\Psi_m(t)|, \quad (19)$$

where

$$|\Psi_m(t)\rangle = \exp(-iHt)|\Psi_m(0)\rangle = U(t)|\Psi_m(0)\rangle. \quad (20)$$

The initial state is

$$|\Psi_m(0)\rangle = |\psi(0)\rangle|\phi_m\rangle.$$

The evolution operator $U(t)$ can be evaluated by different methods. In Ref. 9, they use a unique analytical operator technique. Here, we apply an efficient numerical algorithm based on polynomial schemes^{27–29} into this problem. The method used in this calculation is the Laguerre polynomial expansion method we proposed in Ref. 27, which is pretty well suited to many quantum systems, open or closed, and can give accurate result in a much smaller computation load. More precisely, the evolution operator $U(t)$ is expanded in terms of the Laguerre polynomial of the Hamiltonian as

$$U(t) = \left(\frac{1}{1+it} \right)^{\alpha+1} \sum_{k=0}^{\infty} \left(\frac{it}{1+it} \right)^k L_k^{\alpha}(H).$$

$L_k^{\alpha}(H)$ is one type of Laguerre polynomials³⁰ as a function of H , where α ($-1 < \alpha < \infty$) distinguishes different types of the Laguerre polynomials and k is the order of it. In real calculations, the expansion has to be cut at some value of k_{\max} , which was optimized to be 20 in this study (we have to test out a k_{\max} for the compromise of the numerical stability in the recurrence of the Laguerre polynomial and the speed of calculation). With the largest order of the expansion fixed, the time step t is restricted to some values in order to get accurate results of the evolution operator. At every time step, the accuracy of the results will be confirmed by the test of the numerical stability—whether the trace of the density matrix is 1 with error less than 10^{-12} . For longer times, the evolution can be achieved by more steps. The action of the Laguerre polynomial of Hamiltonian to the states is calculated by recurrence relations of the Laguerre polynomial. The efficiency of this polynomial scheme²⁷ is about nine times as that of the Runge-Kutta algorithm under the same accuracy condition used in Ref. 26. When the states $|\Psi_m(t)\rangle$ are obtained, the density matrix can be obtained by performing a summation in Eq. (19).

Although theoretically we should consider every energy state of the single-mode bath field, $M \rightarrow \infty$, the contributions of the high-energy states $|\phi_m\rangle$, $m > m_C$ (m_C is a cutoff to the

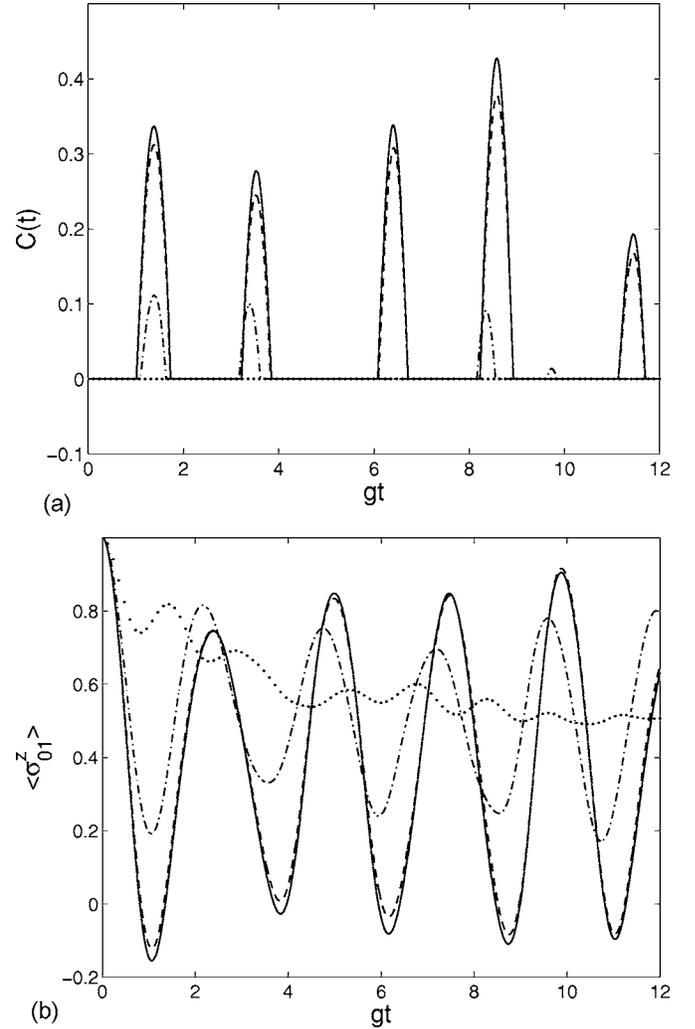


FIG. 1. Time evolution for (a) concurrence and (b) the moment of spin 01 from an initial two-qubit state of $|\psi(0)\rangle = |11\rangle$ at different values of anisotropic parameter: $\gamma=0$ (solid curve), $\gamma=0.2$ (dashed curve), $\gamma=0.6$ (dot dashed curve), and $\gamma=1.0$ (dotted curve). Other parameters are $\mu_0=2g$, $g_0=g$, and $T=g$.

spin-bath eigenstates) are found to be neglectable due to their very tiny weight value ω_m as long as the temperature is finite. That is to say, the M in Eqs. (16)–(19) could be changed to m_C . Then, we use the following equation in real calculation:

$$\rho(t) = \sum_{m=1}^{m_C} \omega_m |\Psi_m(t)\rangle\langle\Psi_m(t)|. \quad (21)$$

After obtaining the density matrix of the whole system, the reduced density matrix is calculated by a partial trace operation to $\rho(t)$, which trace out the degrees of freedom of the environment,

$$\rho_S(t) = \text{Tr}_B[\rho(t)]. \quad (22)$$

For the model of this paper, $\rho_S = |\psi\rangle\langle\psi|$ is the density matrix of the open subsystem consists of two separate spins, which can be expressed as a 4×4 matrix in the Hilbert space of the subsystem spanned by the orthonormal vectors $|00\rangle$, $|01\rangle$,

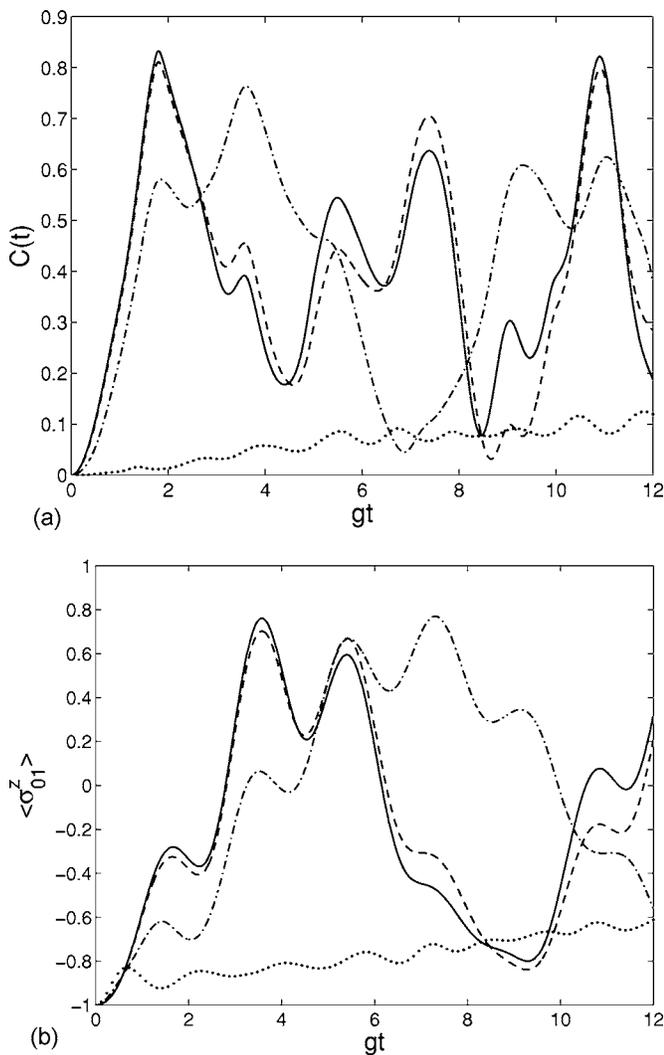


FIG. 2. Time evolution for (a) concurrence and (b) the moment of spin 01 from an initial two-qubit state of $|\psi(0)\rangle=|01\rangle$ at different values of anisotropic parameter: $\gamma=0$ (solid curve), $\gamma=0.2$ (dashed curve), $\gamma=0.6$ (dot-dashed curve), and $\gamma=1.0$ (dotted curve). Other parameters are $\mu_0=2g$, $g_0=g$, and $T=g$.

$|10\rangle$, and $|11\rangle$. The most general form of an initial pure state of the two-qubit system is

$$|\psi(0)\rangle = \alpha|00\rangle + \beta|11\rangle + \gamma|01\rangle + \delta|10\rangle, \quad (23)$$

with

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1. \quad (24)$$

IV. NUMERICAL SIMULATION RESULTS AND DISCUSSIONS

When the reduced density matrix is determined, any physical quantities of the subsystem can be readily found out. In the following, we will discuss three important physical quantities of the subsystem which reflect the decoherence speed, the entanglement degree, and the fidelity of the subsystem state. These quantities are (i) the moment of spin 01

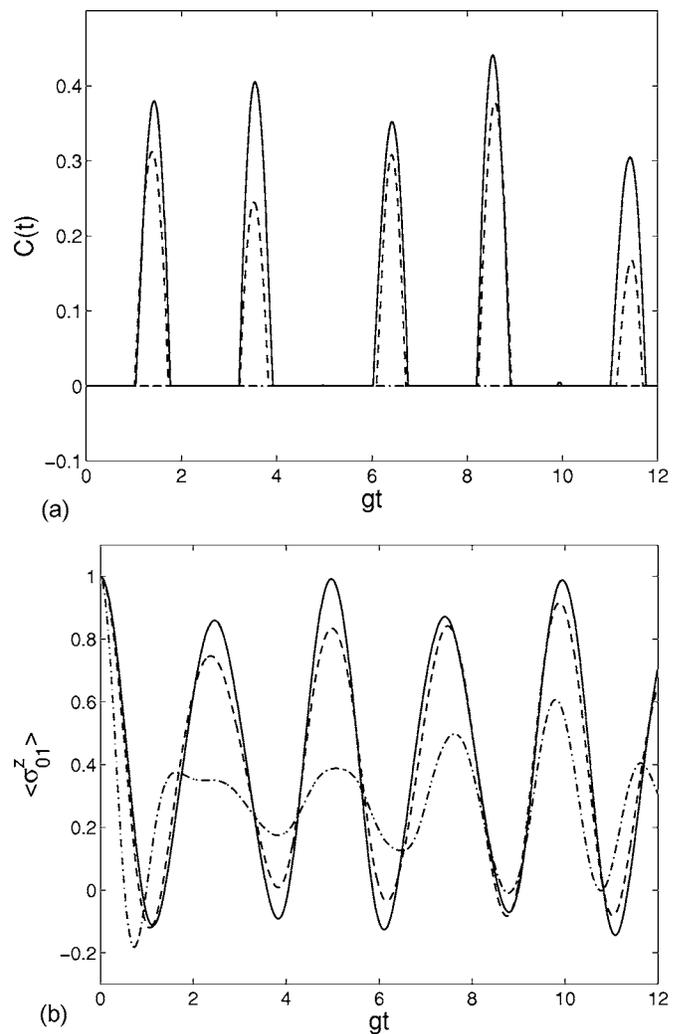


FIG. 3. Time evolution for (a) concurrence and (b) the moment of spin 01 from an initial two-qubit state of $|\psi(0)\rangle=|11\rangle$ at different values of temperature: $T=0.2g$ (solid curve), $T=g$ (dashed curve), and $T=5g$ (dot-dashed curve). Other parameters are $\mu_0=2g$, $g_0=g$, and $\gamma=0.2$.

(here, we choose the first spin $\langle\sigma_{01}^z\rangle$ which demonstrates the decoherence rate of the system), (ii) the concurrence^{31,32} for the two spins of the open subsystem (the concurrence of the two spin-1/2 system is an indicator of their intraentanglement, which is defined as³¹

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \quad (25)$$

where λ_i are the square roots of the eigenvalues of the product matrix $\rho_S(\sigma^y \otimes \sigma^y)\rho_S^*(\sigma^y \otimes \sigma^y)$ in decreasing order), and (iii) the fidelity,³³ which is defined as

$$Fd(t) = \text{Tr}_S[\rho_{\text{ideal}}(t)\rho(t)]. \quad (26)$$

$\rho_{\text{ideal}}(t)$ represents the pure-state evolution of the subsystem under H_S only, without interaction with the environment. The fidelity is a measure for decoherence and depends on ρ_{ideal} , and is equal to 1 only if the time dependent density matrix $\rho(t)$ is equal to $\rho_{\text{ideal}}(t)$. The corresponding results and dis-

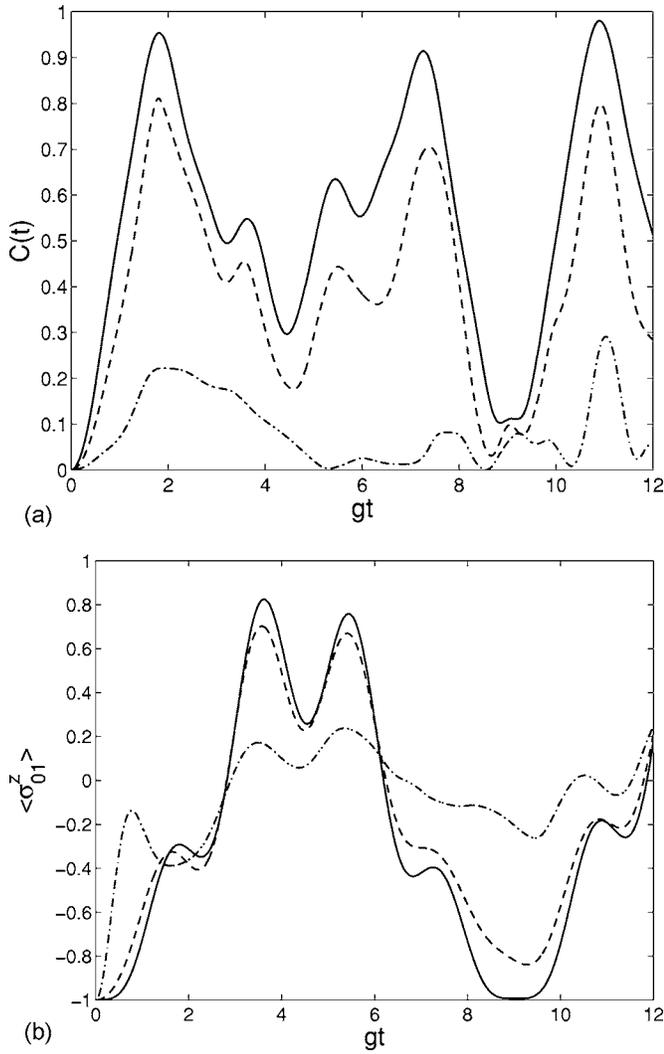


FIG. 4. Time evolution for (a) concurrence and (b) the moment of spin 01 from an initial two-qubit state of $|\psi(0)\rangle=|01\rangle$ at different values of temperature: $T=0.2g$ (solid curve), $T=g$ (dashed curve), and $T=5g$ (dot dashed curve). Other parameters are $\mu_0=2g$, $g_0=g$, and $\gamma=0.2$.

ussions are divided to two subsections according to different initial states.

For the product states, the present paper focuses on the entanglement generation by the spin bath but does not involve the revival of the initial state for $C(t=0)=0$. Thus, in the Sec. IV A, we give out the dynamics of concurrence and σ_{01}^z . For the Bell states, since the system can evolve to a completely different state from the initial one and has the same concurrence $C(t)>0$, we should give out the evolution of concurrence and fidelity in Sec. IV B.

A. Product states

First, we show the evolution of concurrence and the z component moment as functions of anisotropic parameter γ from two initial product states $|11\rangle$ (see Fig. 1) and $|10\rangle$ (see Fig. 2). It is obvious that the entanglement between the two subsystem qubits can arise from the interaction with the bath.

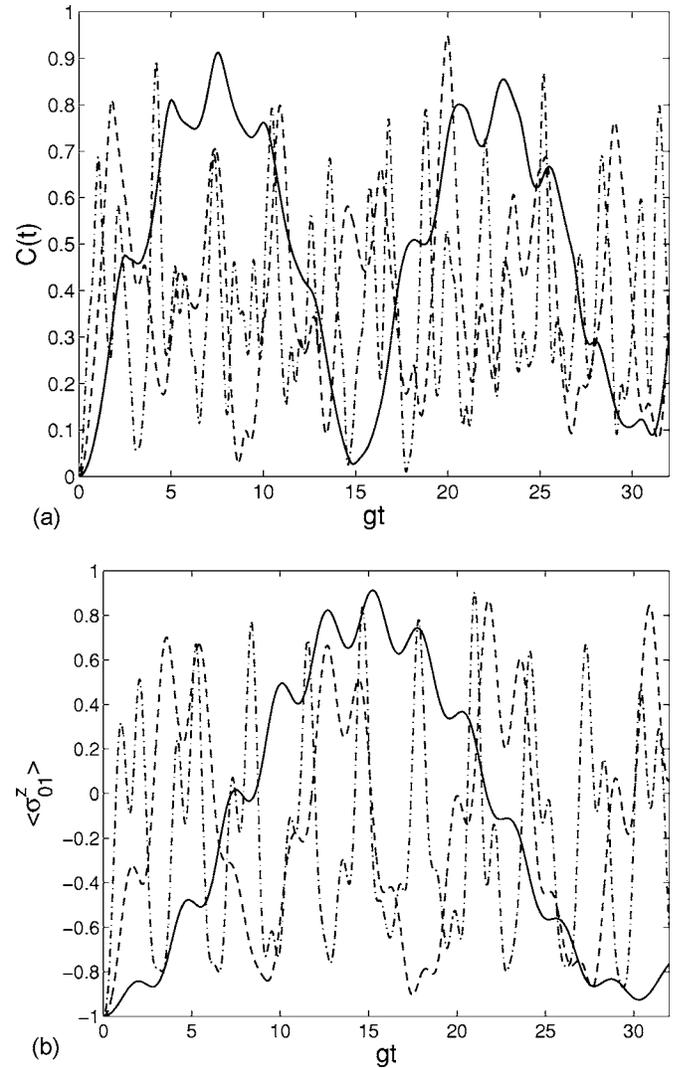


FIG. 5. Time evolution for (a) concurrence and (b) the moment of spin 01 from an initial two-qubit state of $|\psi(0)\rangle=|01\rangle$ at different values of coupling strength between subsystem and bath: $g_0=0.5g$ (solid curve), $g_0=g$ (dashed curve), and $g_0=2g$ (dot dashed curve). Other parameters are $\mu_0=2g$, $\gamma=0.2$, and $T=g$.

Yet, this kind of effect of the spin bath is decreased with increasing the anisotropic parameter γ . The above variation depends on the initial states: with $|\psi(0)\rangle=|11\rangle$ [Fig. 1(a)] when $\gamma>0.87$, the concurrence of the two qubits will be always kept zero as initialed; with $|\psi(0)\rangle=|01\rangle$ [Fig. 2(a)], the entanglement can always be created to some extent; and if γ approaches to 0 (the isotropic case), the concurrence can increase as high as 0.8 over some periods of oscillations. In Figs. 1(b) and 2(b), with γ increasing, the oscillation amplitudes of the curves become smaller and smaller, which means that the coherence of the subsystem approaches the state of being lost. For the initial state $|11\rangle$, after the first spin flip (the sign of σ_{01}^z changes from positive to negative) for $\gamma\geq 0.6$, it cannot flip again. However, for $|\psi(0)\rangle=|10\rangle$, the spin can flip after a period of time even for $\gamma=0.6$. Therefore, the increase of entanglement depends sensitively on the anisotropic parameter.

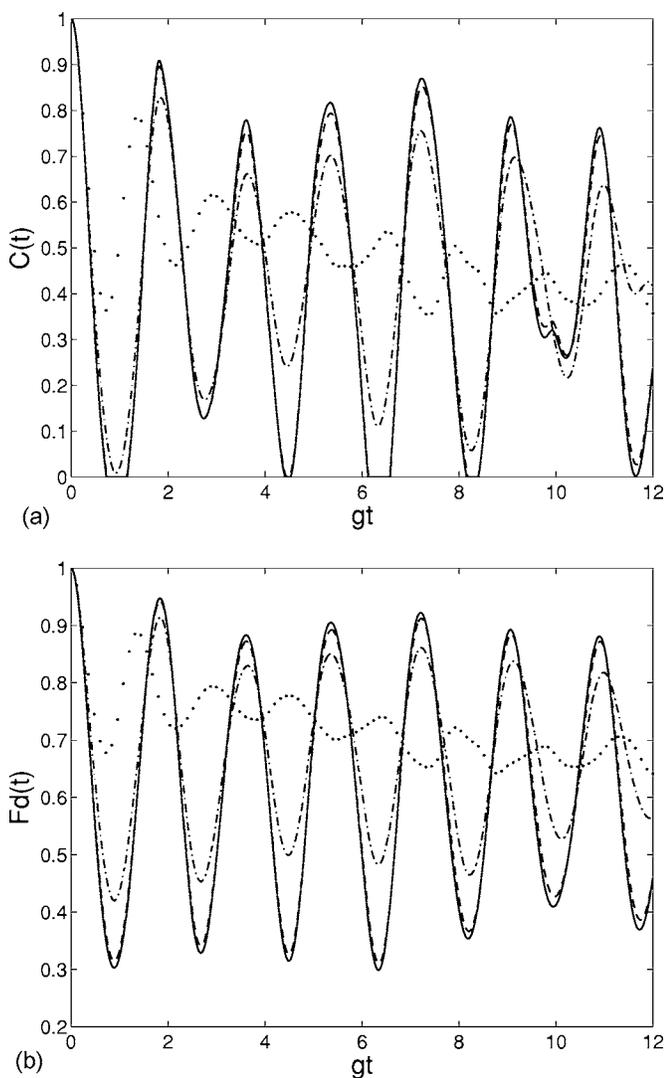


FIG. 6. Time evolution for (a) concurrence and (b) fidelity from an initial two-qubit state of $|\psi(0)\rangle=1/\sqrt{2}(|10\rangle+|01\rangle)$ at different values of anisotropic parameter: $\gamma=0$ (solid curve), $\gamma=0.2$ (dashed curve), $\gamma=0.6$ (dot dashed curve), and $\gamma=1.0$ (dotted curve). Other parameters are $\mu_0=2g$, $g_0=g$, and $T=g$.

The bath is in a thermal equilibrium state at different temperatures the effect of which is shown in Figs. 3 and 4. In these two figures, the anisotropic parameter γ is kept as 0.2. We can find that (i) at a very low temperature, $\sigma_{01}^z(t)$ displays a nearly periodical oscillation, which is identical with the two-photon resonance of two two-level atoms in a cavity, and the subsystem entanglement can be raised to a comparative degree; (ii) with increasing temperature, the oscillation amplitudes of the curves are damped due to the thermal bath. For the concurrence, $C(t)\rightarrow 0$ means to approach a most separated state [to see the dot-dashed curve in Figs. 3(a) and 4(a)]. For σ_{01}^z , it means the degeneration of its magnetic moment [to see the dot-dashed curve in Figs. 3(b) and 4(b)]. Therefore, it is clear that the subsystem loses its memory faster as the temperature increases.

In Figs. 1(a) and 2(a), we can find that the entanglement between the two initial separated spins can be generated with the help of the single-mode thermal bosonic bath field. As-

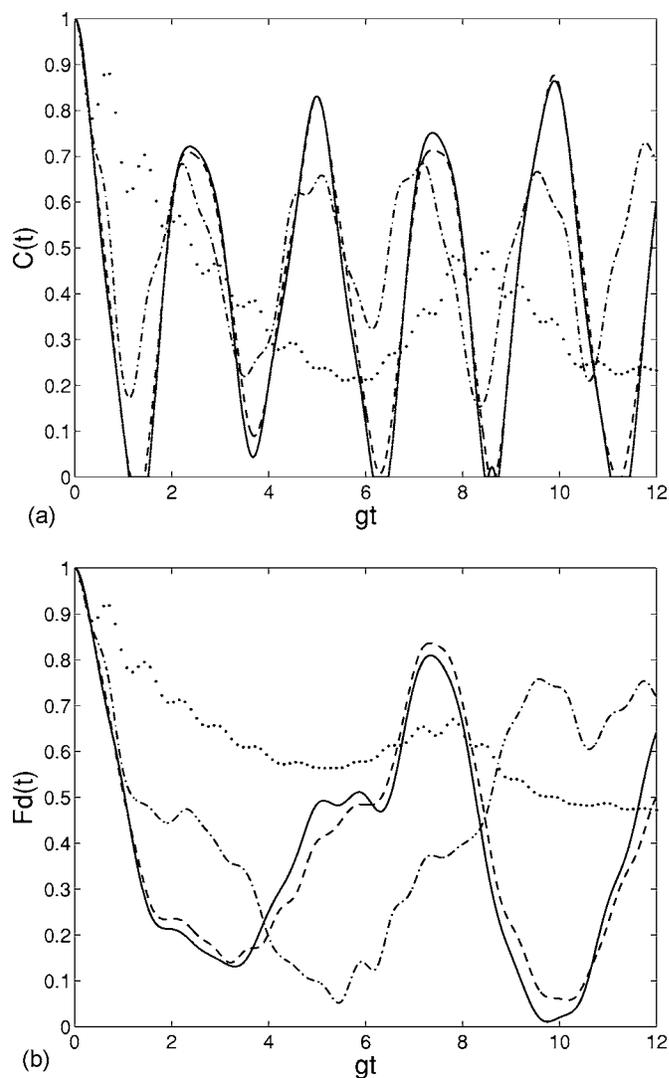


FIG. 7. Time evolution for (a) concurrence and (b) fidelity from an initial two-qubit state of $|\psi(0)\rangle=1/\sqrt{2}(|11\rangle+|00\rangle)$ at different values of anisotropic parameter: $\gamma=0$ (solid curve), $\gamma=0.2$ (dashed curve), $\gamma=0.6$ (dot dashed curve), and $\gamma=1.0$ (dotted curve). Other parameters are $\mu_0=2g$, $g_0=g$, and $T=g$.

sume that the system is initially prepared in $|01\rangle$ (or $|10\rangle$), a pure state. On one hand, when the interaction between the system and spin bath is turned on, one spin could drop from the excited state and simultaneously the other spin could jump absorbing the boson just emitted by the former. This process can induce the correlation of two spins. On the other hand, it also evolves into a mixed state resorting to the bosons provided by the single-mode boson field. Thus, the coupling between the system and its environment leads to the entanglement between two initial separated qubits.

Then, we keep the bath at a moderate temperature $T=1g$ to find out the effect of the coupling strength g_0 in Fig. 5. At a smaller value $g_0=0.2g$, the weak interaction with the bath will make both the concurrence and σ_{01}^z display a pseudoperiodical behavior; on the contrary, at a larger value $g_0=5g$, their dynamics is too strongly disturbed by the bath to be utilized. Thus, in real applications, the coupling between the subsystem and the spin bath should be reduced.

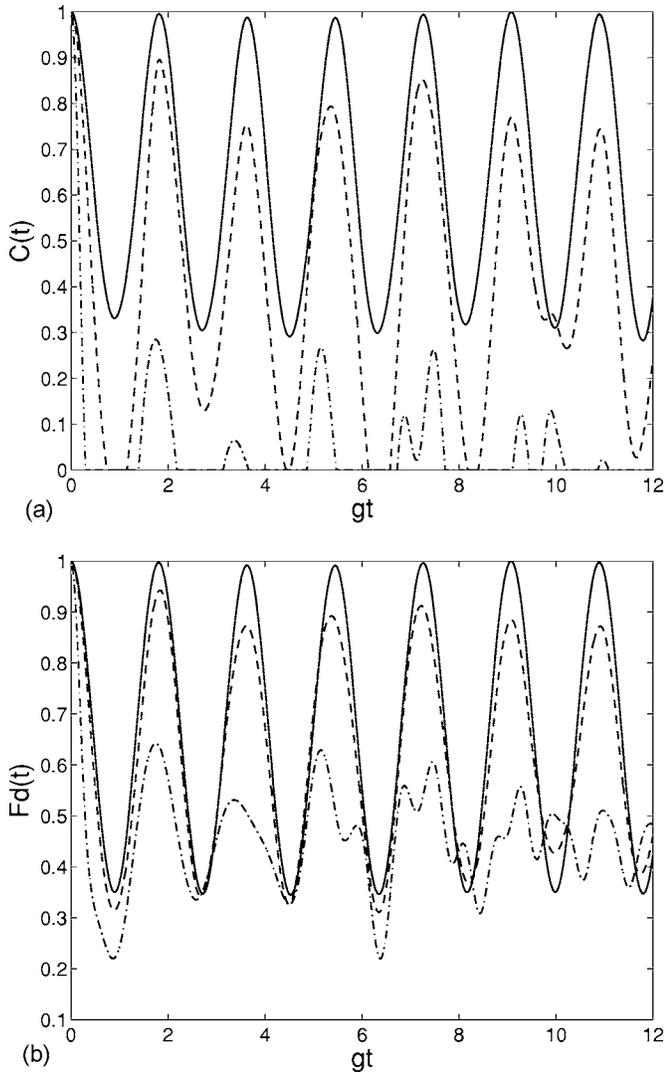


FIG. 8. Time evolution for (a) concurrence and (b) fidelity from an initial two-qubit state of $|\psi(0)\rangle=1/\sqrt{2}(|10\rangle+|01\rangle)$ at different values of temperature: $T=0.2g$ (solid curve), $T=g$ (dashed curve), and $T=5g$ (dot dashed curve). Other parameters are $\mu_0=2g$, $g_0=g$, and $\gamma=0.2$.

B. Bell states

In the cases that the subsystem is prepared as a most entangled state $C=1$ (Bell states), the anisotropic parameter γ still makes an important effect on the time evolutions of concurrence and the σ_{01}^z . When $\psi(0)=1/\sqrt{2}(|10\rangle+|01\rangle)$, the concurrence [see Fig. 6(a)] of subsystem is always revived to $C\approx 0.8$ after some time of oscillation at small value of γ . The results can be proved by the revival fidelity of the subsystem in Fig. 6(b): at every summit, the state mainly consists of its initial state. However, an interesting phenomena is found by the comparison of Fig. 7(a) with Fig. 7(b). It is noticed that two summits disappear in the interval of $0.0 < gt < 8.0$. Thus, we analyze the states at the first three summits in Fig. 7(a). It is found that at $gt=2.448$, the most component of the subsystem state is $1/\sqrt{2}(|11\rangle-|00\rangle)$; at $gt=4.960$, the state of the two qubits is very near to a combination of $1/\sqrt{2}(|11\rangle-i|00\rangle)$ and $1/\sqrt{2}(|11\rangle-|00\rangle)$; at gt

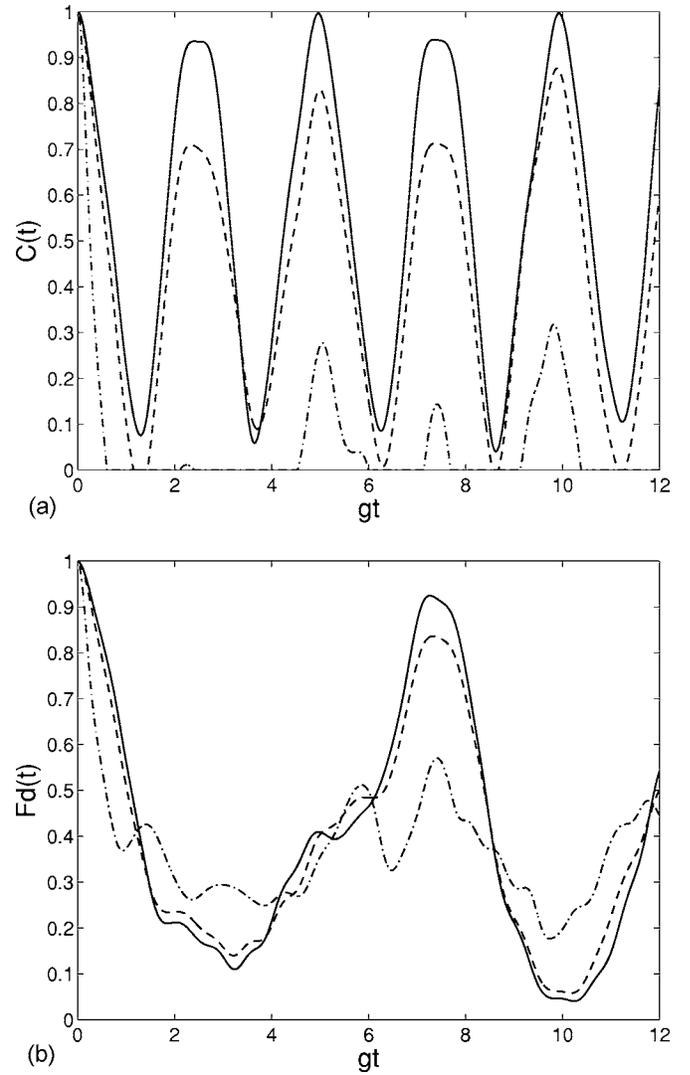


FIG. 9. Time evolution for (a) concurrence and (b) fidelity from an initial two-qubit state of $|\psi(0)\rangle=1/\sqrt{2}(|11\rangle+|00\rangle)$ at different values of temperature: $T=0.2g$ (solid curve), $T=g$ (dashed curve), and $T=5g$ (dot dashed curve). Other parameters are $\mu_0=2g$, $g_0=g$, and $\gamma=0.2$.

$=7.480$, the main part of the state comes back to its initial state. So the concurrence cannot determine the concrete state of the subsystem in the present case. Even if the concurrence can be restored, the state is not always the same as the initial one. Only the combination of the concurrence and the fidelity gives the information of real state evolution.

In Figs. 8 and 9, we plot the dynamics behavior of the concurrence and fidelity at different temperatures. When the temperature is as low as $T=0.2g$, both cases of $\psi(0)=1/\sqrt{2}(|10\rangle+|01\rangle)$ and $\psi(0)=1/\sqrt{2}(|11\rangle+|00\rangle)$ display a periodical oscillation; the concurrence can always nearly restore its initial value. However, for the former case, the dynamics of fidelity is synchronous with that of concurrence; for the latter, the revival of the concurrence does not always mean that of the state. From the viewpoint of the definition of fidelity [Eq. (26)], it is partly due to the system part of the Hamiltonian [Eq. (2)]. In the special case of the Bell state $1/\sqrt{2}(|10\rangle+|01\rangle)$, $\rho_{ideal}(t)$ is identical to the spins initial den-

sity matrix (it is an eigenstate of H_S), while for $\psi(0) = 1/\sqrt{2}(|11\rangle + |00\rangle)$ $\rho_{ideal}(t)$ is not in the same condition. It is further proved that the properties of the dynamics should be determined by the combination of concurrence and fidelity.

V. CONCLUSION

We have studied the dynamics evolution of two separated qubit spins in a bath consisted of infinite spins in a quantum anisotropic Heisenberg XY model. The bath can be treated effectively as a single pseudospin of $N/2$ spin degree. After the Holstein-Primakoff transformation, it will further be considered as a single-mode boson at the thermodynamic limit. The pair of qubits with no direct interaction which served as a quantum open subsystem is initially prepared in a product state or a Bell state. Then, the concurrence of the two qubits,

the z component of one of the subsystem spins and the fidelity of the subsystem can be determined by a novel polynomial scheme during the temporal evolution. It is found that (i) larger anisotropic parameter γ can help to maintain the initial state of the two qubits; (ii) the bath at higher temperature plays a strong destroy effect on the entanglement and coherence of the subsystem, and so does the one with strong interaction g_0 ; and (iii) the dynamics of the subsystem is dependent on the initial state and in some special cases, only the concurrence is not sufficient to judge the revival of the subsystem.

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